Spectral balancing techniques Application to CDMA and UWB signaling

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***Abstract*—This paper presents techniques for generating orthogonal bases of signals with jointly optimized spectra, in the sense that they are made as close as possible. To this end, we propose new criteria, the minimization of which leads to signals with close energy inside a set of prescribed subbands. We present a first algorithm that performs this spectrum balancing and that we apply to Walsh-Hadamard codes. As an example, we build balanced Walsh codes in CDMA comunications. Alternatively, we propose another way to perform spectrum balancing in two steps and that makes use of joint diagonalization techniques. As an example application, we show how balancing of Prolate Spheroidal Wave Functions (PSWFs) can be used to build orthogonal families of brief impulses with flat spectra that are of potential interest for Ultra Wide Band (UWB) communications.**

Keywords orthogonal signaling bases, spectrum balancing, PSWF, Walsh-Hadamard, scrambling, CDMA.

I. Introduction

A few studies have been carried out to build orthogonal signals with flat spectrum. Several of these studies are based on the invariance property of Hadamard matrices with respect to orthogonal transforms. In particular, [19] and [18] consider permutations of rows of the Walsh matrix code, the codes being given by the columns of the matrix. For these methods, flat spectrum is just a mean property, obtained when considering average spectra over code permutations.

Alternatively, from white noise sequences with flat spectra, orthonormality can be achieved via singular value decomposition [3]. Another interesting technique that enables better control of spectral shape consists of splitting the spectral bandwidth of signals into a set of subbands of interest. In each subband, the Fourier transfoms of the sequences are chosen as orthogonal Walsh codes with fixed amplitudes [2]. Proceeding so in each subband yields orthogonal signals in the Fourier domain and thus in the time domain. Note however that with these approaches the shape of the signal in the time domain is not controlled.

In this paper we are interested in building orthogonal signals with similar spectra that belong to some fixed vector space. This will enable to control properties such as signals duration or shape. We shall provide example of their interest for CDMA and UWB communications.

In a CDMA (Code Division Multiple Access) context [21], users transmit simultaneously and inside the same frequency band. They are distinguished thanks to distinct signaling codes. Often, Walsh codes are considered for multiusers spread spectrum communications. Walsh codes of given length show very variable spectra and thus they fail to achieve an homogeneous robustness of all users signaling against multipath fading that occurs during transmission. Classicaly, users signals are whitenned through the use of a scrambling sequence which consists of a sequence with long period that is multiplied, chip by chip, with users' spread data [20]. Scrambling also enables neighbouring base stations insulation in mobile communication networks.

In radiolinks, synchronization of scrambling sequences between base stations and mobiles is not much a problem. But it can be a problem in applications such as an acoustic underwater CDMA communication [11], because low propagation speed results in propagation delays that can correspond to thousands of symbols.

In such difficult situations, we shall not consider complex scrambling code synchronization. Rather, we propose to build orthogonal families of codes made of spreading sequences with flat spectra inside the sequences bandwidth. In addition, we would like to be able to build large sets of such signaling bases, for using distinct ones in neighbouring basestations and/or to be able to change codes during the communications of a given basestation, for instance for robustness against communication interception.

In order to build such codes, starting from a given othonormal code family, we propose to transform it by means of an orthogonal transform. This orthogonal transform is built by minimizing jointly the mean squared errors among energies of all transformed sequences inside fixed subbands that form a partition of the whole sequences bandwidth. Flatness will result from spectrum averaging among the elements of the family. We shall call this algorithm SBA (Spectral Balancing Algorithm).

This technique enables building arbitrarily large number of bases of spectrally balanced orthogonal codes. This is achieved by changing the initialization of the algorithm that we describe in the paper. In particular, distinct bases can be considered for neighbouring bases stations in replacement of scrambling sequences. In addition, for a given basestation it is also possible to change the codes family during transmission. Finally, base station insulation , spectrum whiteness of transmitted signals and data protection that are achieved by scrambling can also be obtained through balanced sequences generation.

We shall consider balancing of CDMA sequences. Without loss of generality, balancing of CDMA codes will be studied for Walsh codes. We shall see that the corresponding balanced sequences exhibit several nice suitable properties such as low auto-correlation and cross-correlation peaks and good multiuser detection BER performance in asynchronous transmissions.

We shall also consider an alternative balancing technique. Here the idea consists first in a joint diagonalization of a set of matrices that yields spectrally maximaly separated spectra of signals. Then, we recombine these signals by using binary orthogonal weightings that can be supplied for instance by Walsh codes. This yields approximately balanced ortogonal signals. Two implementations of this idea that we shall refer to as Spectral Unbalancing-Balancing Algorithm (SUBA) are proposed and denoted by SUBA1 and SUBA2 respectively.

As an example of application, we propose to build highly concentrated waveforms with flat spectrum. These waveforms can be useful for example for UWB signaling where short impulses are emitted [12]. Here, we propose building such waveforms from a set of Prolate Spheroidal Wave Functions (PSWFs) [17], [16].

The remainder of the paper is organized as follows. In section II, we show how energies of an orthogonal family of signals can be equalized simultaneously over a set of frequency subintervals thanks to an orthogonal matrix transform, preserving thus orthogonality of transformed signals. This matrix is obtained through an iterative minimizing algorithm. In section III, we consider Walsh codes balancing. Simulations show that spectrum whitenning achieved by balancing yields good correlation properties of balanced sequences, resulting thus in improved performance of multiuser asynchronous communications. In section IV, the alternative SUBA algorithms are presented and an example of short signals with flat spectra is supplied.

II. Spectrum balancing of an orthonormal family of signals

In this section, we present an orthogonal transform that enables transforming an orthonormal family of signals into another orthonormal family, the elements of which have about the same energy in prescribed frequency intervals. It was introduced first in [7] for energy balancing in a single interval and in [8] for its generalization to jointly equalize energies of a set of signals in a prescribed set of frequency intervals.

Let us denote by $\mathbf{v}_1,\dots,\mathbf{v}_L$ an initial family of sampled orthonormal signals, with $\mathbf{v}_n=(\mathbf{v}_{n1},\dots,\mathbf{v}_{nN})^T$. The energy of $\mathbf{v}_n$ inside a given frequency interval, say $B=[f_1,f_2]$, is given by

$$\begin{aligned} E_B(\mathbf{v}_n) &= \int_{f_1}^{f_2} |\sum_{a=1}^{N} \mathbf{v}_{na} e^{-2i\pi f a}|^2 df \\ &= \sum_{a,b=1}^{N} \mathbf{v}_{na}\mathbf{v}_{nb}^* e^{i\pi(f_1+f_2)(b-a)} \\ &\qquad \times \frac{\sin\pi(f_2-f_1)(b-a)}{\pi(b-a)}, \\ &= \textstyle\sum_{a,b=1}^{N} \mathbf{v}_{na}\mathbf{v}_{nb}^* \mathbf{S}_{ba}^B, \end{aligned} \tag{1}$$

where $\mathbf{S}^B$ is the matrix with general term $\mathbf{S}_{ba}^B = e^{i\pi(f_1+f_2)(b-a)}\sin(\pi(f_2-f_1)(b-a))/(\pi(b-a))$.

Considering Eq. (1), we define a set of matrices $\{\mathbf{S}_k\}_{k=0,K-1}$ associated with a partition $\{B_k\}_{k=0,K-1}$ of the frequency support of signals. For real valued signals, spectra are even functions, letting $[-KF,KF]$ denote the bandwidth of signals $\mathbf{v}_1,\dots,\mathbf{v}_L$, we can take frequency subbands in the form

$$B_k=[(-k-1)F,(-k+1)F]\cup[(k-1)F,(k+1)F]. \tag{2}$$

Then, corresponding matrices $\mathbf{S}_k$ are written as

$$\begin{aligned} \mathbf{S}_{ab}^0 &= \frac{\sin 2\pi F(a-b)}{\pi(a-b)}, \\ &\text{and, for } k=1,\dots,K-1, \\ \mathbf{S}_{ab}^k &= 2\cos(2\pi kF(a-b))\times\frac{\sin 2\pi F(a-b)}{\pi(a-b)}, \end{aligned} \tag{3}$$

where $\mathbf{S}_{ab}^k$ is a compact form for $[\mathbf{S}_k]_{ab}$. Although extension to the complex case is straightforward, for this paper we restrict ourself to the case of real valued signals.

A. Balancing algorithm

We shall denote by $\mathbf{U}$ the orthogonal transform applied to the signals matrix $\mathbf{V}=[\mathbf{v}_1,\dots,\mathbf{v}_L]$ and we shall note $\mathbf{M}_k=\mathbf{U}^T(\mathbf{V}^T\mathbf{S}_k\mathbf{V})\mathbf{U}$, for $k=0,\dots,K-1$. Diagonal entries of $\mathbf{M}_k$ represent the energies of the signals given by the columns of the matrix $\mathbf{VU}$ that lie inside B_k. Our goal is to build a matrix $\mathbf{U}$ such that the diagonal parts of all matrices $(\mathbf{M}_k)_{k=0,\dots,K-1}$ become as close as possible. This will be achieved by successive updatings of $\mathbf{U}$ by means of Givens rotations. The update $\mathbf{U}\to\mathbf{U}\mathbf{R}^{ab}(\theta)^T$ of $\mathbf{U}$ amounts to the update $\mathbf{M}_k\to\mathbf{R}^{ab}(\theta)\mathbf{M}_k\mathbf{R}^{ab}(\theta)^T$ of $\mathbf{M}^k$. In order to jointly equalize diagonal terms of $\mathbf{M}_k$, we can choose θ such that it is a solution of the following minimization problem:

$$\begin{aligned} \theta = \arg\min_\phi \textstyle\sum_{k=0}^{K-1} &|[\mathbf{R}^{ab}(\phi)^T\mathbf{M}_k\mathbf{R}^{ab}(\phi)]_{aa} \\ &- [\mathbf{R}^{ab}(\phi)^T\mathbf{M}_k\mathbf{R}^{ab}(\phi)]_{bb}|^2, \end{aligned} \tag{4}$$

the minimum of which is of the form

$$\theta=\frac{1}{4}\arctan\left(\frac{2\sum_{k=0}^{K-1}(\mathbf{M}_{ab}^k+\mathbf{M}_{ba}^k)(\mathbf{M}_{aa}^k-\mathbf{M}_{bb}^k)}{\sum_{k=0}^{K-1}(\mathbf{M}_{ab}^k+\mathbf{M}_{ba}^k)^2-(\mathbf{M}_{aa}^k-\mathbf{M}_{bb}^k)^2}\right)+\frac{n\pi}{4}, \tag{5}$$

where $n = 0, 1, 2$ or 3. The optimum value for n can be obtained by checking which of the four possible values 0, 1, 2 or 3 achieves the minimum. In practice, it appears that after a few rotation updates the optimum n is always 0, because θ becomes small. It can be checked that taking $n = 0$ at any iteration of the algorithm does not modify significantly its behavior. Thus, for the sake of simplicity we assume that it is always 0.

On another hand, we have checked that the term $(\mathbf{M}^k_{aa} - \mathbf{M}^k_{bb})^2$ in the denominator of the $\arctan(.)$ function in Eq. (5) is a source of unstability. Since it should become close to zero at convergence ($\mathbf{M}^k_{aa} \approx \mathbf{M}^k_{bb}$), we set it to 0 from the beginning of the algorithm. Then we obtain a spectrum balancing algorithm that is summarized in Table II. We shall cal it Spectral Balancing Algorithm (SBA). One can observe that this algorithm resorts to ideas quite similar to those developed for joint diagonalization of matrices [5], [1].

TABLE I
SBA (SPECTRAL BALANCING ALGORITHM).

- Set $\mathbf{U} = \mathbf{U}_0$ with $\mathbf{U}_0^T \mathbf{U}_0 = \mathbf{I}$,
$\mathbf{M}_k = \mathbf{U}^T (\mathbf{V}^T \mathbf{S}_k \mathbf{V}) \mathbf{U}$, $\mathbf{M}_k^- = \mathbf{0}$ $(k = 0, \ldots, K-1$
- Iterations:
while $\sum_{k=0}^{K-1} \| \mathbf{M}_k - \mathbf{M}_k^- \| \geq \varepsilon$, $\quad (\varepsilon \ll 1)$
$\quad \mathbf{M}_k^- = \mathbf{M}_k, \ (k = 0, \ldots, K-1)$
$\quad$ **loop** $a = 1 \to L-1$
$\quad\quad$ **loop** $b = a+1 \to L$

$$\theta = \frac{1}{4} \arctan \left(\frac{2 \sum_{k=0}^{K-1} (\mathbf{M}^k_{ab} + \mathbf{M}^k_{ba})(\mathbf{M}^k_{aa} - \mathbf{M}^k_{bb})}{\sum_{k=0}^{K-1} (\mathbf{M}^k_{ab} + \mathbf{M}^k_{ba})^2} \right)$$

$$\mathbf{M}_k = \mathbf{R}^{(a,b)}(\theta)\ \mathbf{M}_k\ \mathbf{R}^{(a,b)}(-\theta), \ (k = 0, \ldots, K-1)$$

$$\mathbf{U} = \mathbf{U}\, \mathbf{R}^{(a,b)}(-\theta)$$

$\quad\quad$ **end loop** b
$\quad$ **and loop** a
end while
$\mathbf{W} = \mathbf{V}\mathbf{U}$,

III. WALSH CODES BALANCING

It is sometimes interesting to be able to control the vector space the generated sequences lie in. In particular, CDMA systems employ chip-shaped sequences. This structure has given rise to specific processing techniques. For instance, in downlink CDMA systems, the emitted signal is made of multi-user chip symbols shaped by the chip waveform at the transmitter output. At the receiver side, chip rate MMSE (Minimum Mean Square Error) equalizers are an efficient tool for downlink CDMA receivers that exploit this data structure [13]. Clearly, chip level equalization cannot be considered for continuously varying signalings such as those considered in [3] and [2]. This motivates our search for sequences defined at the chip rate.

Since we are looking for signals that are constant over chip intervals, a natural approach is to search for them in the space spanned by the orthogonal Walsh-Hadamard basis. Then, if the sampled signals of this basis are given columnwise in a matrix form, any new orthogonal basis of the vector space is achieved by applying an orthogonal matrix transform on the right hand side. Note that methods in [19] and [18], discussed in the introduction, apply matrix permutations on the left hand side of the matrix of code sequences.

A. Spectral balancing of Walsh sequences

We illustrate Walsh codes spectral balancing with length 8 and 32 in Fig. 1 and 2 respectively. In Fig. 1 and in 2, spectrum balancing is searched over $K = 8$ and $K = 32$ subbands respectively and codes of lengths 8 and 32 chips respectively. In Fig. 2, only 8 randomly chosen codes are plotted among the 32 codes of length 32. Code vectors are sampled with 8 samples per chip. Convergence is achieved after one hundred to several hundred iterations of the main loop of the algorithm. Fig. 3 has been obtained when balancing Walsh codes of length 256 over $K = 256$ frequency intervals, showing thus that long spreading sequences can be produced by the algorithm.

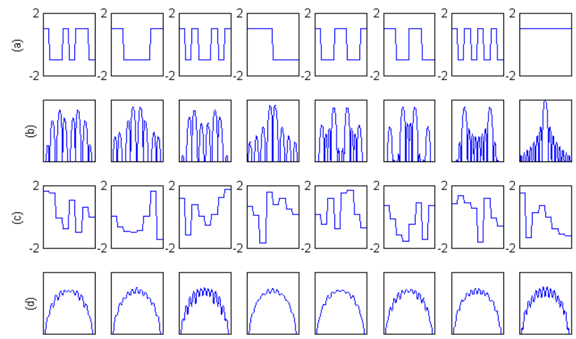

Fig. 1. (a) Walsh codes of length 8 (b) corresponding spectra (c) spectrally balanced codes (d) spectra of balanced codes.

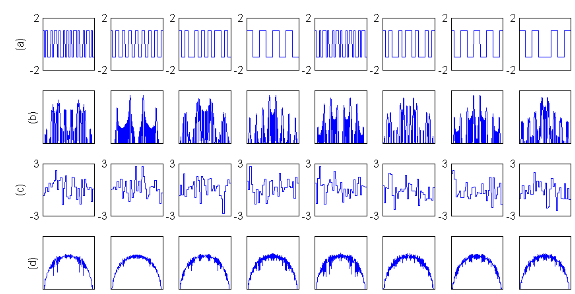

Fig. 2. (a) Walsh codes of length 32 (b) corresponding spectra (c) spectrally balanced codes (d) spectra of balanced codes.

B. Correlation and cross-correlation of sequences

Correlation and cross-correlation properties of codes dictate the performance of a multiuser communication system at high SNR [6]. For simple receivers based on single user matched filter, correlation properties are important in particular for receiver synchronization, while in asynchronous systems cross-correlations of codes limits performance. Thus, we are going to consider these properties and compare them between Walsh codes and balanced codes.

Balanced sequences appear to have useful correlation properties. This is illustrated in Fig. 4. The two first subfigures on the first line in Fig. 4 show superimposed correlation

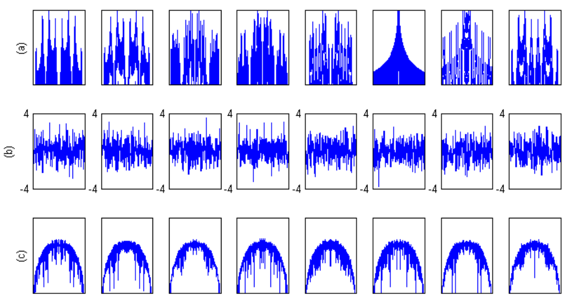

Fig. 3. (a) spectra of a few Walsh codes of length 256 (b) spectrally balanced codes (c) spectra of balanced codes.

functions of the 32 Walsh and balanced codes respectively. Clearly, balanced codes have good autocorrelation properties. In particular, correlation coefficients around the main peak are close to zero. This is an interesting property for CDMA communications, for instance for multipath detection and estimation, but also for using such codes in applications such as synchronization or localization with radars or positioning systems [14].

Since Walsh codes have high correlation sidelobes and high cross-correlation level, we also made a comparison with brute force codes considered in [15]. These codes are obtained by means of an exhaustive search algorithm among codes with good cross-correlation properties. Fig. 4 shows that these codes achieve quite poor correlation performance, even when removing the constant code autocorrelation (the one with triangular shape).

As far as cross correlations are considered, the second line in Fig. 4 shows that both balanced and brute force codes achieve good performance, unlike Walsh codes.

Finally, above results advocate in favor of multilevel balanced codes that can achieve higher correlation performance, at the expense of relaxation of the constant amplitude property.

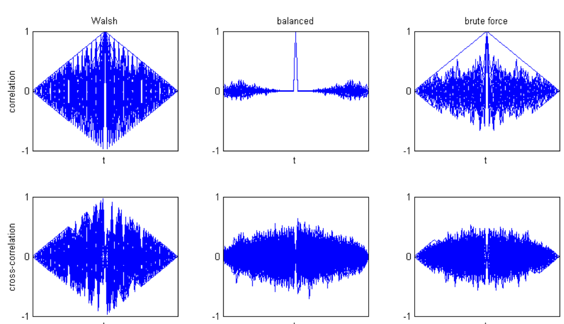

Fig. 4. Superimposed correlations (up) and cross-correlations (down) of Walsh, balanced and brute force codes.

C. Asynchronous transmission

Let us now consider an asynchronous transmissions with the above families of codes and simple matched-filter detection. Transmission on an AWGN (additive white gaussian noise) in the presence of 2 users are presented for Walsh, balanced and brute force codes in Fig. 5. Clearly, balanced and brute force codes achieve similar performance while Walsh codes perform poorly at high SNR. The stars in Fig. 5 represent the performance lower bound for matched filter detectors under the standard Gaussian asumption of interference [6] while the lower curve is the single user performance bound. We see that both balanced and brute force codes reach the bound, proving thus their optimality in terms of the level of interference. Another example is supplied in Fig. 6 for codes of length 256 and 32 users. For this code length, brute force codes are not available in [15]. Balanced codes still show performance closer to the interference lower bound than Walsh codes.

Of course, for a fixed spreading code length, the matched filter receiver performs worse as the number of interfering users increases and decorrelator or MMSE detectors would lead to improved BER curves [21]. However, here we only considered the simpler matched filter receiver to focus on code properties rather than on receiver performance.

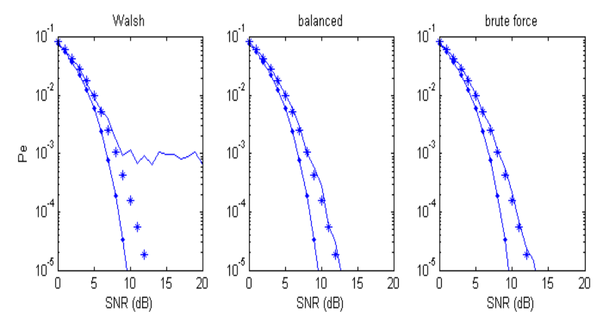

Fig. 5. Asynchronous transmission with Walsh, balanced and brute force codes. Code length = 32 and 2 users. Lower curve: single user BER; Stars: 2 users interference lower bound.

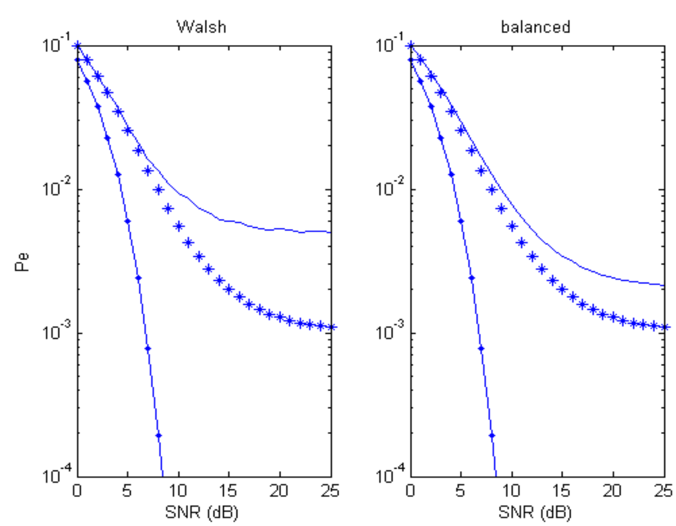

Fig. 6. Asynchronous transmission with Walsh and balanced codes. Code length = 256 and 32 users. Lower curve: single user BER; Stars: 32 users interference lower bound.

IV. An alternative balancing algorithm

A. Maximal unbalancing

In the previous section, we have looked for spectrally maximaly balanced families of signals. Now, we are going to study this problem via an opposite approach consisting in looking to maximaly unbalanced families of signals. This amounts to performing orthogonal transformation of an orthogonal family of vectors that represents initial signals in such way that the new signals have spectral supports as distinct as possible.

Letting as above $\mathbf{M}_k = \mathbf{U}^T(\mathbf{V}^T\mathbf{S}_k\mathbf{V})\mathbf{U}$ and defining $\mathbf{Z} = \mathbf{VU}$, maximum concentration of the energy of the k^{th} column of $\mathbf{Z}$ inside the k^{th} subband amounts to maximizing the energy of $[\mathbf{M}_k]_{kk}$. Jointly maximizing these matrix entries leads to the following optimization problem :

$$\begin{cases} \max_{\mathbf{U}} \sum_{k=1}^{K} [\mathbf{M}_k]_{kk} \\ \mathbf{U}^T\mathbf{U} = \mathbf{I}. \end{cases} \tag{6}$$

Proceeding as in the previous section, we can maximize iteravely the criterion by solving the optimization problem over rotations in two-dimensional subspaces with entries (a, b) and letting $\mathbf{U} \to \mathbf{U}\mathbf{R}^{ab}(\theta)^T$, where $\mathbf{R}^{ab}(\theta)^T$ is the optimum rotation. One can check that for $\mathbf{R}^{ab}(\theta)$ the angle should now be of the form

$$\theta = -\frac{1}{2}\arctan\left(\frac{2([M_a]_{ab} - [M_b]_{ba})}{([M_b]_{aa} - [M_a]_{aa}) - ([M_b]_{bb} - [M_a]_{bb})}\right). \tag{7}$$

We shall call this procedure Spectral Unbalancing Algorithm (SUA).

The result of this procedure is to supply orthogonal signals with spectral supports as much distinct as possible. Going back to our initial problem that amounts to build orthogonal families with balanced spectra, we can benefit from the fact that maximaly unbalanced signals in $\mathbf{Z}$ have roughly disjoint spectra that cover the bandwidth spanned by the columns of $\mathbf{V}$. Denoting by $\mathbf{H}$ an $N \times N$ orthogonal matrix with entries ± 1, such as the Walsh-Hadamard orthogonal matrix, and $\mathbf{W} = \mathbf{ZH}$, then, from the properties of $\mathbf{Z}$ and $\mathbf{H}$, the columns of $\mathbf{W}$ are orthogonal and should have spectra with close amplitude. All this can be summarized in the procedure summerized in Table II and named spectral unbalancing-balancing algorithm (SUBA1).

TABLE II
1 ALGORITHM.

- Set $\mathbf{U} = \mathbf{U}_0$ with $\mathbf{U}_0^T\mathbf{U}_0 = \mathbf{I}$,
$\mathbf{M}_k = \mathbf{U}^T(\mathbf{V}^T\mathbf{S}_k\mathbf{V})\mathbf{U}$, $\mathbf{M}_k^- = \mathbf{0}$ $(k = 0, \dots, K-1$
- Iterations:
while $\sum_{k=0}^{K-1}[\mathbf{M}_k - \mathbf{M}_k^-]_{kk}^2 \geq \varepsilon$, $\quad (\varepsilon \ll 1)$
  $\mathbf{M}_k^- = \mathbf{M}_k$, $(k = 0, \dots, K-1)$
  loop $a = 1 \to L-1$
    loop $b = a+1 \to L$
      $\theta = -\frac{1}{2}\arctan\left(\frac{2([M_a]_{ab} - [M_b]_{ba})}{([M_b]_{aa} - [M_a]_{aa}) - ([M_b]_{bb} - [M_a]_{bb})}\right)$
      $\mathbf{M}_k = \mathbf{R}^{(a,b)}(\theta)\,\mathbf{M}_k\,\mathbf{R}^{(a,b)}(-\theta)$, $(k = 0, \dots, K-1)$
      $\mathbf{U} = \mathbf{U}\,\mathbf{R}^{(a,b)}(-\theta)$
    end loop b
  and loop a
end while
$\mathbf{W} = \mathbf{VUH}$.

Alternatively, we have defined a procedure called SUBA2 that differs from SUBA in the construction of matrix $\mathbf{Z}$. The matrices $\mathbf{S}_k$ can be seen as matrices of correlation coefficients associated to spectra equal to the index functions of the corresponding frequency intervals B_k. Thus, from Whittle's approximation, the eigenmatrix of all the matrices $\mathbf{S}_k$ is approximately the discrete Fourier transform matrix. In addition, corresponding eigenvalues of $\mathbf{S}_k$ are approximately 1 or 0 depending on whether the frequency of the Fourier approximate eigenvector is in B_k or not. This suggests that performing the approximate joint diagonalization of matrices $\mathbf{M}_k$ should lead to eigenvectors with roughly disjoint spectra. Finally, SUBA2 can be summarized as follows:

TABLE III
SUBA2 ALGORITHM.

- Compute $\mathbf{Q}$, the matrix of joint approximate eigenvectors of matrices $\mathbf{T}_k$ $(k = 0, \dots, K-1)$
- $\mathbf{W} = \mathbf{VQH}$.

In practice, the joint eigenvalue decomposition can be achieved by any standard algorithm. In the simulation part, we have applied the algorithm in [10], that can be found on the web [4]. We have checked on examples that as N increases SUBA2 performs better than SUBA1. In addition, one should note that the number K of frequency intervals must be set to N with SUBA1, while it can be chosen differently with SUBA2, although this has little effect on balancing performance.

B. Example

One may look for signaling functions basis that concentrate all their energy within frequency interval $[-F, F]$ and with most of their energy concentrated in a time interval of length T. The solution of this problem is supplied by Slepians's prolate spheroidal wave functions (PSWFs) basis [17] that consists in a family of orthogonal functions that are solutions of the following integral equation

$$\int_{-T/2}^{T/2} \frac{\sin(\pi F(t-t'))}{\pi(t-t')} v(t)dt = \lambda v(t). \tag{8}$$

Spectrum balancing of Slepian sequences or PSWFs can be of interest in Ultra Wide Band (UWB) communications. Indeed, in UWB, M-ary pulse shape modulation has been proposed and it can be achieved with orthogonal signals such as PSWFs [12]. The spectra of Slepian sequences or PSWFs are slightly shifted upward as order increases. Instead, spectrally balanced pulses have spectra that better occupy the whole bandwidth.

More generally, spectrum balancing could be considered for other UWB orthogonal pulses, such as Gaussian, Hermite or Legendre functions, where elements of the family of increasing order tend to have spectra with energy concentrated at increasing frequencies [9].

In fig. 7, maximum unbalancing of $N = 8$ PSWFs obtained with SUBA1 and SUBA2 respectively are presented. energy concentration in contiguous subbintervals appears clearly in both cases as well as the bandlimited property of the whole set of unbalanced sequences.

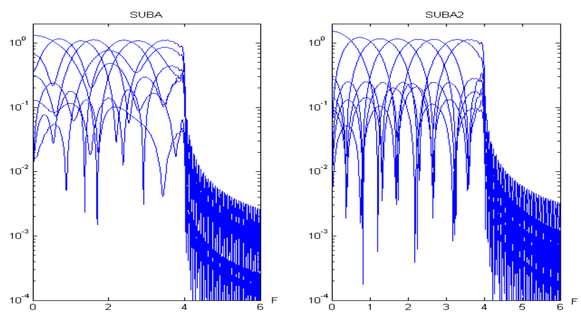

Fig. 7. Spectrum unbalancing of first 8 PSFWs. The spectra of 4 randomly chosen sequences obtained by SUBA1 (left) and SUBA2 (right) are presented.

The second unbalancing procedure yields more regular spectra. For this procedure, SUBA2 algorithm yields the signals and spectra in fig. 8 that show optimal time concentration and flat spectra in the bandwidth.

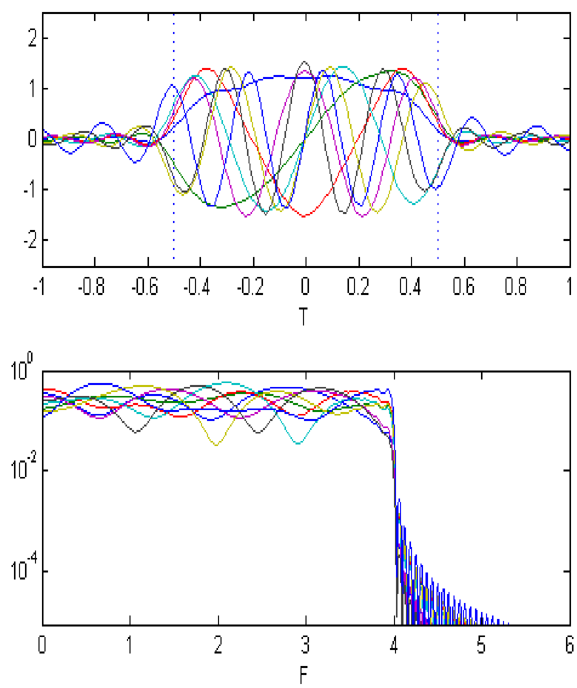

Fig. 8. Balancing of first 8 PSFWs with SUBA2: signals (up) and spectra (down).

V. Conclusion

We have proposed general purpose procedures for deriving spectrally balanced bases of signals in a given signal subspace, approximated as a subspace of $\mathbb{R}^N$. As an example, we have shown how it is possible to build efficient signalization sequences for CDMA multiuser communications that show performance similar in terms of BER to brute force optimized binary sequences. Large numbers of such families of codes can be built thanks to the relaxation upon the constant amplitude constraint. Clearly using such balanced signals in applications such as synchronization or for designing radar waveforms is promising, due in particular to useful correlation properties and the wide variety of waveforms that can be generated. We have also proposed alternative algorithms for spectral balancing of an orthogonal family of signals via maximum spectral unbalancing as an intermediate step. We have shown how bandlimited signals with flat spectra and short duration can be built from this approach.

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